NUMERICAL INVESTIGATION OF FREE CONVECTION FROM AN INSTANTANEOUS HEAT SOURCE IN VISCOUS FLUID

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A formulation is given, as well as results of the numerical solution, of the self-consistent problem of free convection from an instantaneous heat source in a viscous incompressible fluid.

Free convection is considered arising in an unbounded uniform medium which is a viscous incompressible fluid as a result of instantaneous pointlike heat emission. This process can be described, provided the usual simplifications of the convection theory remain valid, by the following system of equations in the cylindrical coordinate system r0z (with symmetry with respect to the z axis):

$$\frac{\partial\omega}{\partial t}-\frac{\partial\psi}{\partial z}\frac{\partial}{\partial r}\left(\frac{\omega}{r}\right)+\frac{\partial\psi}{\partial r}\frac{\partial}{\partial z}\left(\frac{\omega}{r}\right)=-\beta g\frac{\partial T}{\partial r}+ \\ +\nu\left(\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\frac{\omega}{r^2}+\frac{\partial^2\omega}{\partial z^2}\right), \quad (1)$$

$$\frac{\partial^2\psi}{\partial r^2}-\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{\partial^2\psi}{\partial z^2}=-r\omega, \quad (2)$$

$$\frac{\partial T}{\partial t}-\frac{1}{r}\frac{\partial\psi}{\partial z}\frac{\partial T}{\partial r}+\frac{1}{r}\frac{\partial\psi}{\partial r}\frac{\partial T}{\partial z}=a\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right) \quad (3)$$

with the boundary conditions

$$\psi=\omega=\frac{\partial T}{\partial r}=0\,(r=0);\ \psi=\omega=T=0\ (r^2+z^2\to\infty). \quad (4)$$

For the system (1)-(3) the enthalpy conservation holds:

$$2\pi\int_0^\infty\int_{-\infty}^\infty Trdrdz=Q. \quad (5)$$

The problem under consideration is self-consistent. In [1] an attempt was made to obtain an approximate analytic solution by expanding the sought solution in an infinite power series. The obtained solution, however, was only valid for low values of the Gr number. Another attempt at analytical solution in [2] involved a priori simplications of the original equations (1)-(3) based on the boundary-layer theory; one also assumed that it was valid for high values of the Gr number. In the present article the problem is solved numerically.

The characteristic scales of the process were selected in the following manner:

$$L=(2\beta g\Theta)^{1/4}t^{1/2},\ V=\frac{dL}{dt},\ \Theta=\left(\frac{Q}{8\beta^3g^3}\right)^{1/4}t^{-3/2}. \quad (6)$$

New variables can now be introduced: $r'=r/L$, $z'=z/L$, $\psi'=\psi/L^2V$, $\omega'=\omega L/V$, $T'=T/\Theta$, and the original equations (1)-(3) are now transformed into (the primes being omitted)

$$-2\omega-\left(\frac{1}{r}\frac{\partial\psi}{\partial z}+r\right)\frac{\partial\omega}{\partial r}+\left(\frac{1}{r}\frac{\partial\psi}{\partial r}-z\right)\frac{\partial\omega}{\partial z}-\frac{\omega}{r^2}\frac{\partial\psi}{\partial z}= \\ =-2\frac{\partial T}{\partial z}+\left(\frac{2}{\mathrm{Gr}}\right)^{1/2}\left(\frac{\partial^2\omega}{\partial r^2}+\frac{1}{r}\frac{\partial\omega}{\partial r}-\frac{\omega}{r^2}+\frac{\partial^2\omega}{\partial z^2}\right), \quad (7)$$

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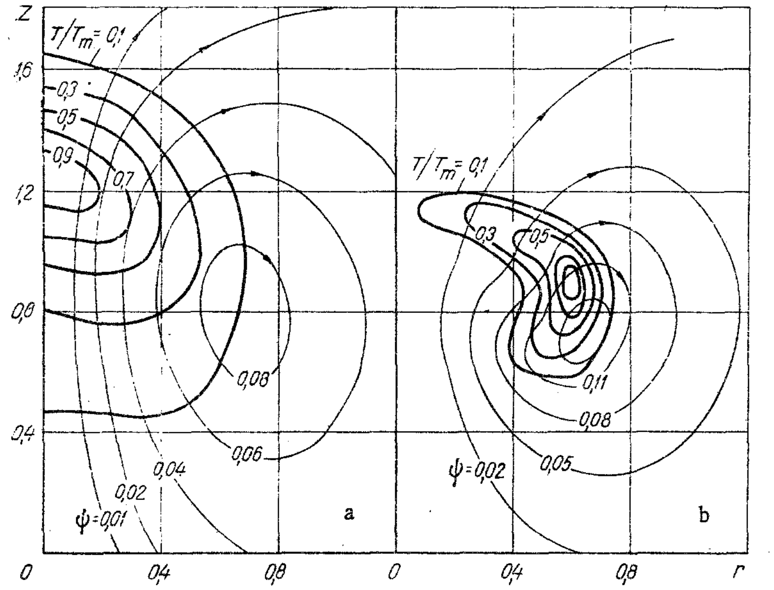

Fig. 1. Distribution of stream function and of temperature in heated bulk for Pr = 1.0 and a) Gr = 200, T_m = 2.12; b) Gr = 20,000, T_m = 4.69

$$-3T-\left(\frac{1}{r}\frac{\partial\psi}{\partial z}+r\right)\frac{\partial T}{\partial r}+\left(\frac{1}{r}\frac{\partial\psi}{\partial r}-z\right)\frac{\partial T}{\partial z}=\left(\frac{2}{\mathrm{Gr}}\right)^{1/2}\frac{1}{\mathrm{Pr}}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right), \tag{8}$$

where $\mathrm{Gr} = \beta g Q/\nu^2$; $\mathrm{Pr} = \nu/\alpha$.

Equation (2) and the boundary conditions (4) expressed in dimensionless variables remain the same, and condition (5) assumes the form

$$2\pi\int_0^\infty\int_{-\infty}^\infty Tr\,dr\,dz = 1. \tag{9}$$

The above equations can also be nominally valid for the averaged flow parameters in the full turbulent state, $\nu_t \gg \nu$, $\alpha_t \gg \alpha$, following the principles of the well-known Prandtl hypothesis for a free boundary layer that $\nu_t = \alpha_1 L_t V_t$ and $\alpha_t = \alpha_2 L_t V_t$ [3]. Here the values α_1 and α_2 characterize the level of the flow turbulence and are related to the initial conditions of the convective element forming. Since in this case one has $L_t \sim Q^{1/4}t^{1/2}$ and $V_t \sim Q^{1/4}\cdot t^{-1/2}$, therefore ν_t and α_t are independent of time. Consequently, the corresponding coordinate transformations result in equations which are the same as (7) and (8), the only difference being that the Gr and Pr numbers are replaced by their turbulence analogs equal to $2/\alpha_1^2$ and α_1/α_2, respectively.

Equations (7) and (8) together with (2), (4), and (9) were solved numerically by the adjustment method for various values of Gr = 200-20,000 and Pr = 0.5-7.0 for which the laminar state of flow is maintained. The analysis of whether it is correct to transfer the boundary conditions (4) from infinity to the finite boundary of the computation domain can be found in [3] as well as the description of the numerical procedure. The computations were carried out on a rectangular difference grid of 42 × 21 cells and with the coordinate steps $\Delta r = \Delta z = 0.08$.

In Fig. 1 the self-consistent distributions of ψ and T are shown computed for the values Gr = 200 or 20,000 for Pr = 1.0, and in Fig. 2 the same distributions are shown for Gr = 1240 and Pr = 0.5 or 2.0. For fairly small values of Gr the numerical solution yields values which are qualitatively in agreement with the approximate solution of [1] although, strictly speaking, the latter is valid for the values of Gr not only exceeding 10. Its characteristic feature lies in its nearly spherical form of the isothermic surfaces of the heated bulk with some concavity in its rear part (Fig. 1a). The same configuration was also observed in experiments with turbulent heated bulk in cases with a considerable initial level of turbulence [4-6].

With the Gr number increasing due to the effect of the convective transfer of turbulence and of heat exceeding that of the diffusive transfer, the concavity in the rear part of the

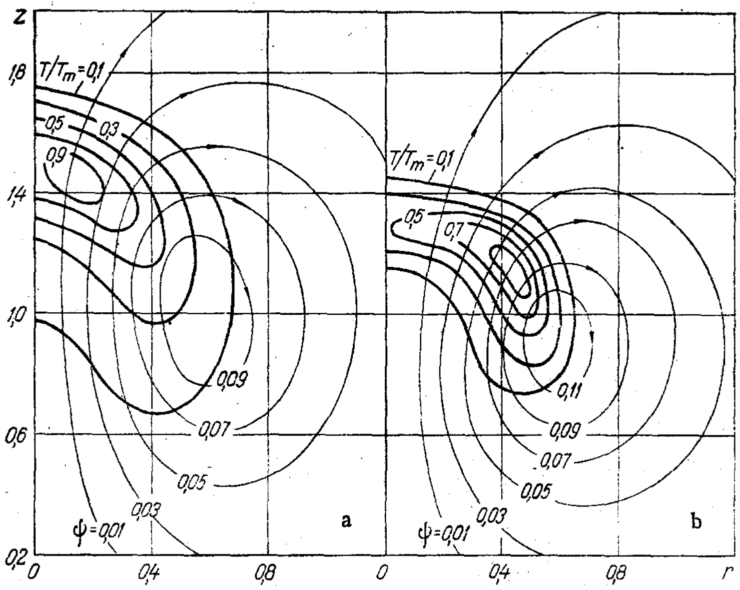

Fig. 2. Distribution of stream function and of temperature in heated bulk for Gr = 1240 and a) Pr = 0.5, T_m = 2.39; b) Pr = 2.0, T_m = 3.64.

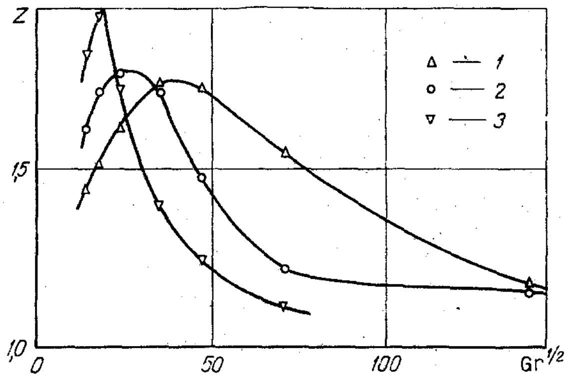

Fig. 3. Relative height of thermal lift of the heated bulk against Gr and Pr numbers: 1) Pr = 0.5; 2) 1.0; 3) 2.0.

heated bulk increases, and on the periphery a closed zone is also formed in which the temperature exceeds its highest value on the symmetry axis. As a result of the increased heat concentration on the periphery the heated bulk assumes a characteristic mushroom shape, that is, it changes into a torus-shaped configuration (Fig. 1b). Higher Pr have a similar effect on the corresponding distributions in the heated bulk as that due to higher Gr (Fig. 2a, b).

The mushroom and toruslike shapes obtained by numerical computations were also observed in experiments with laminar heated bulks for moderately large Gr [7] and for heated bulks with a fairly low turbulence level [8]. However, the results of [2] are not confirmed by the above experimental results and our computations.

In Fig. 3 the highest ordinate L of the heated bulk referred to length Z is shown against the Gr and Pr numbers. One can see from the diagrams that after some initial growth the relative height of the rise starts to decline for higher Gr values. This, at first sight, paradoxical effect was predicted on the basis of theoretical considerations and was also observed experimentally [9]. A higher Pr shows a similar effect.

Unfortunately, only qualitative comparisons can be made between the results obtained by numerical computations and the experimental data, owing to lack of information necessary for quantitative estimates. The latter is due to the relatively long time needed for attaining the self-consistent state for higher Gr values and also to the special features of the process on the not self-consistent portion (in particular, due to considerable heat losses [7]);

primarily, however, it is due to the difficulties in obtaining reliable experimental data for this kind of unstable flow. The latter determines the efficiency of numerical methods for investigating the process at this stage.

NOTATION

r, z, axes of cylindrical coordinate system; ψ, stream function; ω, vortex; T, excessive temperature in relation to unperturbed surrounding medium; t, time; g, gravitational acceleration; β, volume expansion coefficient; ν, a, coefficient of kinematic viscosity and thermal diffusivity; ν_t, a_t, turbulence analogs of the respective coefficients; Q, power of instantaneous heat generation; L, V, and Θ, length, velocity, and temperature scales, respectively; L_t, V_t, and Θ_t, the same, but for turbulent flow; Gr and Pr, Grashof and Prandtl numbers; α_1, α_2, proportionality coefficients; Z, highest ordinate of frontal boundary of heated bulk (on the isotherm $T = 0.1T_m$); T_m, highest value of excessive temperature.

LITERATURE CITED

1. B. R. Morton, J. Fluid Mech., 9, 107 (1960).
2. V. M. Mal'bakhov, Izv. Akad. Nauk SSSR, Fiz. Atmos. Okeana, 8, 683 (1972).
3. F. B. Kaplanskii and A. M. Épshtein, Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1976).
4. R. S. Scorer, J. Fluid Mech., 2, 583 (1957).
5. B. Woodward, Q. J. R. Meteoral. Soc., 85, 144 (1959).
6. J. M. Richards, J. Fluid Mech., 11, 369 (1961).
7. D. J. Shlien and D. W. Thompson, J. Fluid Mech., 72, 35 (1975).
8. T. Fohl, J. Appl. Phys., 38, 4097 (1968).
9. J. S. Turner, Proc. R. Soc., A239, 61 (1957).

INVESTIGATION OF HEAT TRANSFER OF FREE CONVECTION IN BOUNDED VOLUME WITH HEATING FROM ABOVE

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The convection coefficient is found against the Rayleigh number in a bounded volume with heating from above.

The effect of free convection in the case of heating from above has been studied relatively little [1]. It was established in [2-4] that free-convection streams arise in a bounded volume in the case of nonuniform heating of the upper wall. In [3], Nu as a function of Ra was found experimentally in the interval 10^2-10^5 in accordance with which Nu increases from 1 to 1.2.

The neck is the basic "thermal bridge" in vessels for cryogenic fluids with a wide neck and in cryostats. It is established in the present article that convection streams can arise in the neck due to heating from above from the surrounding medium leading to a considerable increase of the heat influx to a cryogenic fluid.

The investigations of free-convection heat transfer were carried out on models in the form of vertical cylindrical vessels or tubes in the 60-200-mm-diameter range and the 90-180-mm-height range. The stand arrangement is shown in Fig. 1. The cylindrical vessel 1 whose lower part is filled with liquid nitrogen is enclosed by a protective chamber 2. The evaporating nitrogen leaves the vessel at the top. Heat transfer between the vessel wall and the exiting cold gas results in a decrease of the heat flux along the wall [5]. In addition to forced convection in the neck, free convection can also arise which produces heat flux through the gas from the upper cap toward the liquid.

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